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Learning Obstacle Student in Factoring Quadratic Form

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Abstract. The purpose of this study is to understand students' mistakes in using the cross method to factorize the squared form. Eight quadratic form factoring is given to students to obtain student learning barrier data. The data were analysed in accordance with the phase of the cross method, followed by the interview to validate the alleged category of learning obstacle types that occurred in each student. The results of this study indicate that the obstacle learning that many students experience using the cross method is didactical obstacle which is one example of the mistake that student's fact squared form by not involving variable x.

1. Introduction

Factoring using cross method has been used by students in Hong Kong, Singapore and Malaysia [1-3]. But most students there have errors in applying the cross method [1-3]. The student's error in factoring is due to inexperienced and adept students in factoring algebraic expressions [3]. Theoretically, this error arises because the main focus of the cross method is to look for the right combination on factoring the x^2-term & the constant, which means the cross method is more intensive in involving operational thinking [2]. Although theoretically the students' inability to use the cross method is understood because it is more likely to involve operational thinking than structural thinking. But in order to develop more structural factoring designs, this theoretical understanding is not enough to provide information on the causes what student failure in learning factoring using the cross method. An analysis is needed to understand more about the causes of student failure when using the cross method. Therefore, by analyzing students' errors based on the obstacle learning category proposed by Brousseau [4], it is expected to become information to develop the factoring learning design and can be used as a teaching guide.

2. Method

A junior high school teacher teaches factoring material using cross method for 26 students of 8th grade. After completing the factoring material, 8 questions about quadratic forms are tested for 40 minutes. The student's answers are they evaluated, all wrong answers are grouped by error category. Each student in each error category is then interviewed to determine the learning obstacle category.

2.1. Instrument

This research uses 8 quadratic form factoring problems. The problem is developed by the researcher based on two categories of quadratic type shapes that are squared form with coefficient x^2 which is equal to one and squared form with coefficient x^2 which is not equal to one. The issue has also been discussed with three school math teachers to see the difficulty of the given problem. The results of the discussion are summarized in Table 1.

Table 1. Assessment Results on Problem Difficulty Level

Evaluator	Question number							
	1	2	3	4	5	6	7	8
First	Easy	medium	medium	medium	medium	medium	difficult	difficult
Second	Easy	Easy	difficult	medium	Easy	Easy	difficult	medium
Third	Easy	Easy	medium	medium	Easy	Easy	medium	difficult

Quadratic form problems are also designed so that students think in getting a combination of quadratic factor factors and the constant terms of the squared form. The difficulty in determining and compressing the appropriate factors is designed by using negative integers for the coefficients of some terms in the squared form. By looking at the key answer in Table 2, we can see the squared form having the writable factor in three different ways, the quadratic form no 4. Problem No. 7 is a perfect squared form, so that the student who understands this can use the nature $(a + b)^2 = a^2 + 2ab + b^2$ in factoring. Problem No. 8 is designed by swapping the position of the constants and quadratic terms.

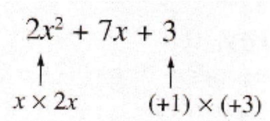
Tabel 2. Problems

No	Quadratic form & factors
1	$x^2 + 7x + 12 = (x + 4)(x + 3)$
2	$x^2 - x - 12 = (x - 4)(x + 3)$
3	$6x^2 + 5x - 6 = (3x - 2)(2x + 3)$
4	$2x^2 + 8x + 6 = (2x + 2)(x + 3) = 2(x + 1)(x + 3) = (x + 1)(2x + 6)$
5	$x^2 - 9x + 14 = (x - 7)(x - 2)$
6	$x^2 + 8x - 9 = (x - 1)(x + 9)$
7	$9x^2 - 12x + 4 = (3x - 2)(3x - 2) = (3x - 2)^2$
8	$3 - x - 2x^2 = (3 + 2x)(1 - x)$

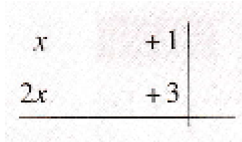
2.2. The process of learning cross method

Before learning cross method was introduced to the students. Teachers first introduce the trial and error method for the quadratic form $ax^2 + bx + c$ with a = 1. After the method has been taught, begin the teacher to introduce the square $ax^2 + bx + x$ with a ≠ 1. By giving some examples the teacher asserts that this method is not efficient enough in solving a question whose quadratic form has a coefficient a ≠ 1. So, the teacher introduced the cross method as the solution of the problem. When beginning to demonstrate the use of cross method, the teacher does so in accordance with the procedure as in figure 1. However, for the following examples, the teacher manipulates it by not involving the variable x in the first step. The teacher argues that by not involving the variable x in the method of factoring the method. Students will more easily do factoring. Learning is continued by giving examples and exercises to students from textbooks used. After one week of learning passed, the students were given 8 questions of the squared form contained in Table 2. After two days of corrections to the results of the answers. Some students who experience errors in answering questions are met for interviews.

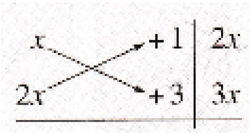
Step 1
Determine the possible factors of the terms in x^2 and also those of the constant term.



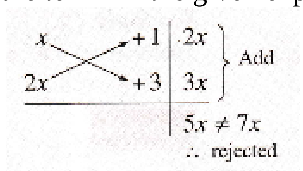
Step 2
Write the factors vertically as shown.



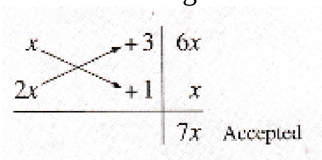
Step 3
Cross-multiply the factors and write the product in the last column.



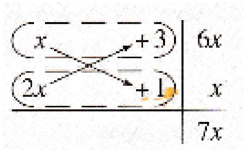
Step 4
Add the last comumn. Reject the work if the sum is not equal to the termx in the given expression.



Step 5
Exchange place for 1 and 3. Cross – multiply and add the last column again.



Step 6
As step 5 is accepted, the factors of the quadratic expression are those "circled"



$\therefore 2x^2 + 7x + 3 = (x + 3)(2x + 1)$

Figure 1. Factoring method in certain text book [5]

2.3. Learning Obstacle

The analysis of students' mistakes in factoring the quadratic form is done by referring to the obstacle learning category consisting of ontogenic obstacle, didactic obstacle, and epistemological obstacle [4]. Ontogenic obstacles occur because students' cognitive abilities have not been able to understand the design presented by the teacher. This can happen because of learning incompetence that occurred before. So, while doing the process of assimilation and accommodation during the learning process caused a structured error. Didactical obstacle occurs because of the instructional design that is delivered and the learning process undertaken. While epistemological obstacles occur when students are unable to use an understanding of the concepts presented in different contexts. The context in question is a different form of problem.

3. Result and Discussion

3.1. Error Analysis by Problem Item

The initial error analysis presented in Diagram 1 shows that question 8 is the most difficult problem for students to be factored into. The question has been predicted by two weighing teachers as a difficult question for students to do. But the question of no 7 predicted by two weighing teachers as a difficult problem to factorize is, in fact, only a student who has experienced a mistake in factoring the matter. Problems 3 and 4 are both hardest to work with. This is already predicted by the weighing teacher, who says that this problem is difficult to be factored because the two questions have more than one quadratic coefficient. Question No. 6 and 2 is a matter of moderate category because the two questions require students to determine two factors of the constant term which are negative numbers. So, it seems that many students are also difficult to solve the problem

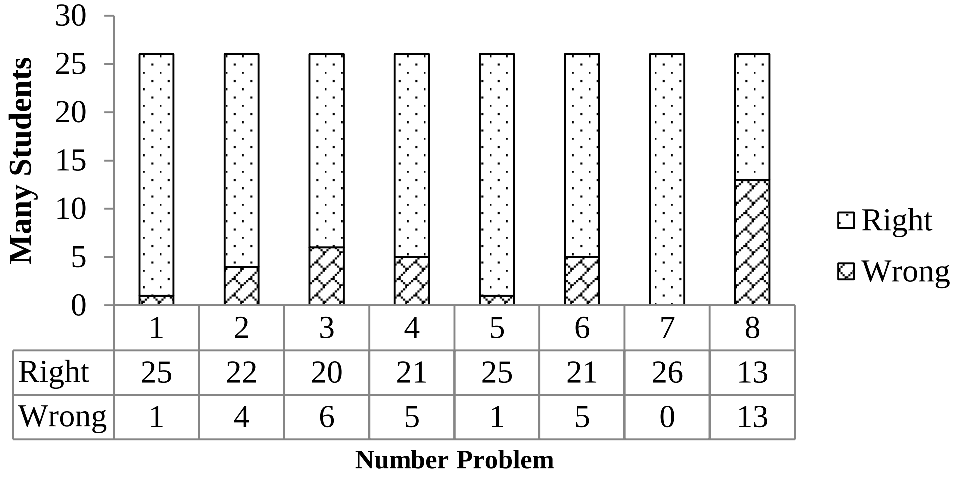


Figure 2. Frequency Error

3.2. 3.2. Error Category Analysis

After a wrong answer was analysed, eleven categories of errors were made by the students. The categories of student error in factoring are presented in Table 4. Of the various types of errors students make, there are two types of errors that have the highest percentage of incorrectly summing the different numbers of signs and wrongs in pairing these factors into the multiplication of two linear factors. Whereas when paying attention to the phase error occurs (Figure 3). It turns out that many mistakes made by students occur in stages 4, 5 and 6 which is the stage of examining and writing the multiplication of two linear factors in the procedure of cross method.

Table 3. Error Category

No	Category	Frequency	Percentage
1	Using another wrong procedure	5	12%
2	Wrong in transform, do not alter the (+) and (-) correctly	2	5%
3	Determine the third term factor by not considering the second term factor	2	5%
4	Does not multiply its factors	1	2%
5	Incorrect in multiplying factors of the first term and third term	1	2%
6	Do not add up the multiplication of the factors	3	7%
7	Incorrect in summing positive and negative numbers	10	24%
8	Not writing the final answer	1	2%
9	Incorrect in pairing these factors into the multiplication of two linear factors	9	22%
10	Incorrectly copy marks (-) and (+)	1	2%
11	Incorrectly placing x value	6	15%

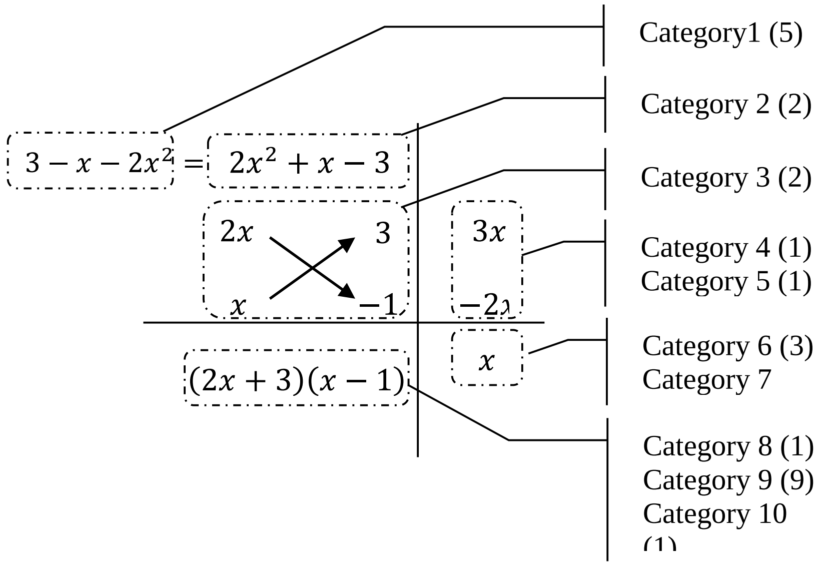


Figure 3. Error Category Illustration

When looking at the error category column, many students experience obstacles in using the cross method in step 3, stage 4 and stage 6. Although slightly different from the initial findings of the analysis. But this shows what Patrick meant by the student making mistakes in stage 3 because the student assumes that the cross method is as perfunctory in doing the multiplication [2].

3.3. Learning Obstacle Based Error Analysis

After conducting interviews with students who make mistakes, obtained two types of learning obstacle that is didactical obstacle and epistemological obstacle. Here are the results of the analysis to define the type of learning obstacle.

3.3.1. Didactical Obstacle

Didactical obstacle occurs due to two factors. The first factor is the design of the cross method which gives less affirmation of pairs of factors obtained to be arranged into the multiplication of two linear factors. The second factor is the teacher presents the cross-method design by not involving the variable x while running the procedure. Didactical obstacles based on the first factor can occur in some questions. While the didactical obstacle on the second factor only occurs on problem 8.

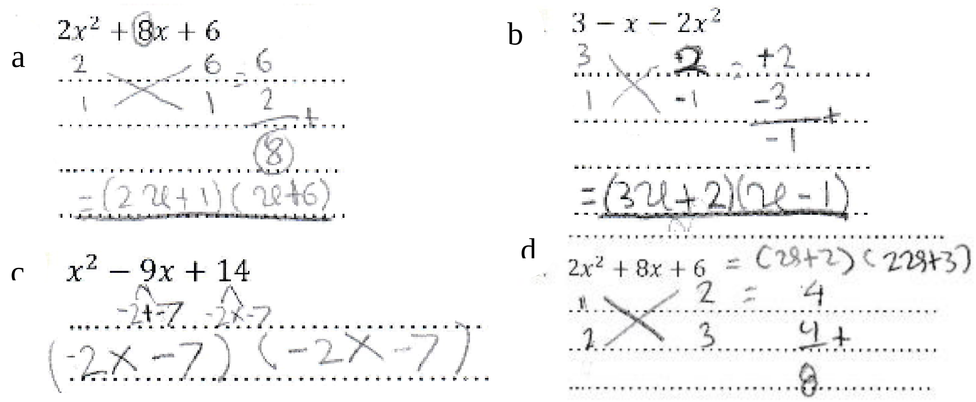


Figure 1. Examples of student answers that include didactical obstacle

In Figure 1a, the student answers incorrectly in pairing factors obtained to be arranged into the multiplication of two linear factors. Students arrange the pair of factors obtained in accordance with the direction of the arrow. Answer 1b, shows the wrong student's answer in writing down the factor that has

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the variable x. Students are accustomed to writing variable x on the first syllable of each linear factor. Thus, the variable x is written side by side with 3 and 1, which should co-exist with 2 and -1.

In Figure 1c, the answer shows the student's memory of the two mixed procedures. When students have got a factor of 14 that amounts to -9. This means that students use the "trial and error" factoring procedure. While in arranging the multiplication of two linear factors. Students treat these factors as four distinct terms, arranged in two linear factors multiplied. This happens because during the lesson, the teacher gives two factoring procedures to complete the quadratic form. In Figure 1d, students break the procedure. Indeed, the sum of the two pairs of multiplied factors must get the result 8. But the students add up 1 + 3 = 4 and 2 + 2 = 4. This seems reasonable because of the number 8. Different results if the factors are multiplied, the number becomes 7. Whereas students should be able to reorder these factors so that 1 × 2 = 2 and 2 × 3 = 6. So, the number is 8.

3.3.2. Epistemological Obstacle

Epistemological obstacle occurs because students are not able to use existing procedures on the type of problem that he has never met. In Figure 3, the student has performed the procedure of cross method. But when you want to write down the multiplication of two linear factors, he is confused how to write it. This is because the squared form that he reports his term order is unusual.

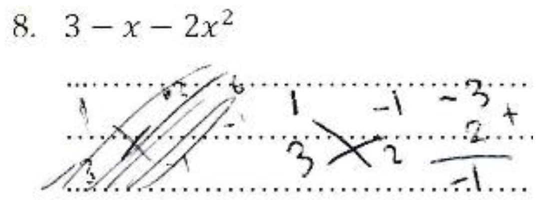

Figure 1. Examples of student answers that include epistemological obstacles

The teacher's opinion that the design of the "cross" methodology would be easier if it did not involve the variable x is wrong. Because, the findings show that many students experience didactical obstacles. Therefore, the factoring design should involve the variable x as a factor of a term in the squared form as a whole. Because the quadratic formwork is done on algebraic operations, not on arithmetic operations. The design of factoring the "cross" method also gives less meaning to the students. The meaningless design of this factoring is due to appear suddenly, not based on previous knowledge. The factoring method of "cross" also uses the arrow direction as a guide to multiply the factors derived from the first and third syllables. The direction of the arrow is often misunderstood as a guide for writing the pair of term to be arranged on the multiplication of two linear factors.

Understanding of the term in the algebraic form is necessary when the student wants to change the unusual square shape form into the general squared form. Understanding of the term as a whole is also needed to be able to determine the exact factor of the term. So, the previous learning about the term and its factors became an important prerequisite in the form of quadratic formation. Some students also exhibit a lack of ability to perform addition operations involving positive numbers and negative numbers. But when the factor in question already involves the variable x. Then the ability of prerequisites that must be fulfilled is able to perform the operation of the addition of different algebraic forms of sign.

The inability of students to use the "cross" method of factoring procedure in factorizing unusual squares gives teachers an opportunity to use unusual squares as a matter of challenge, whose answers are classically discussed.

4. Conclusion

Two obstacle learning is encountered in the quadratic form factor learning. Didactical obstacle occurs because (1) the design of a cross-method methodology taught by a teacher that does not involve variables. (2) the factoring design of the cross method gives less affirmation of the coupled factor pairs into the multiplication of two linear factors. (3) the design of the cross-method is taught after the "trial and error" method, allowing for the mixing of the two procedures of the method. (4) understand procedures with leniency rules. Epistemological obstacle occurs because students are only accustomed to factoring quadratic forms arranged as a general form. So, when the matter is composed with different forms such as the form generally students cannot write the multiplication of two linear factors. Understanding of learning obstacles experienced by students while using cross method factorizing provides an opportunity to design a factoring method that can minimize the learning obstacle experienced by students.

Learning obstacles in factorizing squares can be minimized by improving the method of factoring by taking into account and giving meaning in the multiplication phases between the factors of the quadratic term and the constant term, paying attention to the student's learning completeness while learning the addition operation and the algebraic form involving the negative-marked term so that the student can give pay more attention to the process of checking whether the selected factor is worthy or needs to be replaced, and provides a logical reason for the rules of terms paired in the multiplication of two linear factors.

5. Acknowledgments

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