The Analysis of students’ mathematical reflective and critical thinking ability

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Abstract. This research is aimed to students of mathematics education with the purpose to analyze the ability of higher-order thinking at formal operational stage in mathematics learning. This research was conducted by giving tests that can measure the ability of reflective and critical thinking mathematically and interviewing subjects based on their answers. The results obtained are then analyzed to see the student’s ability level. Research instrument used is a test of reflective and critical thinking ability, and interview with students. The results showed that students already have a good reflective and critical thinking ability in mathematics learning.

1. Introduction
Mathematics is one of the important subjects in education. Mathematics learning is directed to develop (1) the ability of mathematical thinking which includes: understanding, problem solving, reasoning, communication, and mathematical connections; (2) the ability to think critically, as well as an open and objective attitude, as well as (3) disposition of mathematically or habits, and attitude of a high-quality study [1,2]. A process of teaching and learning is considered successful is when learners absorbance against teaching materials being taught reached high achievement, both individually and group and expected behavior in learning objectives have been achieved by learners. So a person’s thinking abilities became one of the milestones for the achievement of learning objectives, especially high level thinking ability as a reflective and critical thinking ability [3, 4]. In learning mathematics are not only taught to just memorize mathematical formulas alone, but must also be able to use mathematics to solve existing problems in the vicinity of their lives.

Reflective thinking is thinking that means, based on the reasons and objectives [5]. Reflective thinking is a future competence that should be taught to students to adapt changes and respond demands of the 21st century [6]. The reflective thought process (reflective thinking) can be described as information or data that is used to respond to, coming from inside (internal), can explain what has been done, realize the mistake and fix it and able to communicate ideas with symbols or images not by direct object [7]. Some indicators to identify the mathematical reflective thinking ability i.e. identify concepts and mathematical formula involved in mathematics questions that are not simple, check the truth of an argument, generalize and analyze the given problems [8].

Critical thinking is thinking ability that involves analysing, synthesising and evaluating the concept, distinguishes between something or data that are relevant and not relevant, identifies and analyzes the assumptions, and accesses the evidence to obtain a truth [9]. The capabilities contained in the critical thinking abilities including the ability give arguments, using the syllogism, inference, perform the evaluation of the conduct, and the ability to create something in the form of new knowledge that has the orisinilitas so that critical thinking ability is a highly mathematical thinking ability necessary to instil
[10]. As for the indicators used to identify the critical thinking ability of students, among others, check out the truth of the argument, statements and process solutions, answers accompanied by reasons, consideration, assess thoroughly, and looking for an alternative [11]. Each of these indicators must be mastered by students to help them to solve the problem either given at school or their lives.

After conducting a study of the literature, we find that there are stages in the process of cognitive development of students. A person who has aged 12 years or more has been at the formal operational stage. The some abilities in formal operational stage according to Piaget’s theory include, (1) the ability to develop abstract thinking capacity, idealistic, and logical, (2) more flexibility in manipulating information (linguistic flexibility), (3) can use symbols to describe any other symbol, (4) use of hypothetical reasoning-deductive, and (5) start thinking about possibilities and think about thinking [12].

Based on the problem background, the researcher conducts the study about the analysis of students’ mathematical reflective and critical thinking ability. This study is conducted with analyze a subject thinking ability at formal operational stage by facilitating the subject to solve mathematics problems that have already identified high level thinking ability that is the ability of reflective and critical thinking and interviewing subject to confirm each solution of given problems. The purpose of this research is to analyze the high order thinking ability of students in the stage of formal operations.

2. Experimental Method
This research used the qualitative method with descriptive-narrative approach. Good data are a record of "what happens" and interpretation which are not based on data that are indeed vacant. The procedures described here are intended to lead researchers through a process of interpretation solidly rooted in data [13]. In this study, qualitative data obtained were described to produce an idea of cognitive ability of the subject researched. This research was conducted at the mathematics education student aged 23 years. The reason chooses the subject according to Piaget’s theory of cognitive development in which adults are on the formal operational stage and already has a high level of thinking ability.

The collection of data in this study using two main instruments, that are main and supporting instruments. The main instrument in this study is a sheet of questions has been compiled on the basis of indicators of the capabilities that will be analyzed. While the instrument of his supporters in the form of interviews with the subject. Interview conducted i.e., face-to-face interview between researcher and the subject. The natural interview which was designed directed to bring up the opinion subject [14].

3. Result and Discussion
The test is given in the form of two mathematics problems that have been validated and are able to measure the ability of reflective and critical thinking of students mathematically. After subject finishes to give answer of the question, researchers conduct interviews with the subject regarding the problems that have been worked on. The first problem that is given is reserved with the level being about relationship rekurensi associated also with material line and sequence numbers. The subject is expected to solve the problem given by making a mathematical model that suits the problem and solve it with a reason of completion. This is a problem on some indicators of ability of reflective and critical thinking mathematically. Indicators that include among others identify concepts or mathematical formula involved in answering the question, accompanied by reasons, consideration, assess thoroughly, and examine the truth of the argument and the process of solution [8, 11].

Here is question No. 1 is presented as follows: On an island, each pair of rabbits gave birth to 2 pairs of rabbits at the time their age begets 1 month and 6 pairs of rabbits again each month started their age 2 months. (a) Can you count the many pairs of rabbits on the island after n months? If you can, calculate. (b) If not, explain the reason you can and fill it with the necessary information until you can count them, then calculate. (c) Check again the truth of your answer. Have you convinced? Give reasons.

The subject takes 12 minutes 50 seconds to finish first problem given and asks for additional time 1 minute 28 seconds to write down the number that is forgotten to render in the answer sheet but he have sought the answer in the graffiti paper. When discussed on the answer, the subject has performed the examination several times until he was sure with the answers. This process is very important in learning mathematics. Because by checking again the arguments and process solutions, then it can be reflective thinking ability of a person is measured [8]. But the reasons given are still less precise. So to make a
form of mathematics, subject follows the steps that he assumes it. As in Piaget called the planning process undertaken by someone in solving problems as hypothetical-deductive reasoning [12]. When the interview took place, the subject admits that he is less focused on the statement in question has been granted. He is more likely to use the thinking based on previous experience and knowledge without paying attention to new problems that have emerged [15]. So to critical thinking on the part of indicator of focuses on a question or issue that is given is still not reached.

The next question relates to the combination of the concept and function of the plant. This problem belongs to a difficult problem. There are five questions asked on this matter. Its questions are deliberately arranged in no way ordinary. Those questions are expected to measure the ability of students’ mathematical critical and reflective thinking. It is aimed at improving the mindset of readers about. The subject is expected to use a combination of concepts and functionality of power plant in the settlement.

Here is a question that is given to the subject: Is it true that there are as many as \( \binom{n}{k} \) how to distribute \( n \) identical balls into \( k \) boxes (\( n \geq k \)), if each box must be filled at least one ball? (a) if true, write down all the ways of distributing 4 identical ball into 2 boxes if every box must be filled at least one ball. (b) If it is not correct, please specify the correct answers/formula. (c) Using the formula you think is true, calculate how many how to distribute 12 identical ball into 3 box if each box must be filled at least one ball. (d) Can you solve the problem (c) using more than one way? If you can, finish them using all ways possible. (e) Are you sure with your answer? Give reasons.

The subject can solve all the problems on question No. 2 within 28 minutes 8 seconds and no additional time is requested. Answers given to the first question on the subject indicates that he is a person active learners, where he was trying to find information to assist his understanding of the problems encountered with the experimental possibilities that may give it a completion [15]. The subject seeks constantly to find confidence over what has been tested. After that he wrote one of the possibilities.

Next on the second points of questions, answer subject is less precise. When the interview took place, the subject gives the reason of the answer and it turns out he made errors before getting the desired formula. This is because it is less focused on workmanship reserved 2. The atmosphere of the time working on the reserved 2 is different when working on problem 1. If the previous is very quiet and there are no distractions, but the next time the place of work question has already started, sound workmanship and building the sound of a ringing phone. The answer to the second question that was less precise answer to the question error resulted in the third. Then the fourth question on the subject say no to resolve the issue before with other means. When all answers are confirmed during the interview, the subject knows the mistakes he did when answering the question of tests and trying to justify it through explanations orally. Observer gives the last question in order to convince the subject thinking. Someone who is capable of critical thinking can provide an explanation of the mathematical models stored in their memory and they don't pour into answers [16]. This is in accordance with the circumstances experienced by the subject while working on the problem. He said less precise answer sheet however he was able to pour the answer through a verbal explanation. So it has been the subject of critical thinking in the face of a given problem.

On the last question posed, the subject gives confidence in his answer based on the concepts that he understand. This indicates that the subject has been reflecting on itself against the answers that she gave. In accordance with the definition of reflective thinking ability that is meaningful thought, based on reason and purpose [17]. So it can be said that the subject has had a reflective thinking ability is good enough and able to account for what he thought was right by giving appropriate reasons of knowledge. To know the ability of reflective and critical thinking of students, we describe one of the answers of the students in Figure 1. In the first part the student is asked to identify concepts or mathematical formula involved in the problem, then students can answer the accompanying reasons and reasoning over the given problem, assess thoroughly, and in part last checked the truth of the argument and the process of solution [8, 11].
4. Conclusion
Based on the research conducted, we can be concluded that (1) the cognitive ability of the subject in the age of formal operations has had a high rate. (2) based on the ability of the indicators examined, subject is already capable of reflective and critical thinking mathematically. (3) this research showed the suitability of results obtained from the subject that was researched with the theory of cognitive development, according to Piaget's formal operations. But in some cases there is still a slight difference that occurs due to the influence of other factors. So, overall the subject has showed the ability of good reflective and critical thinking.

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